

# Our Lady's RC Primary School



## Calculation Policy

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Person Responsible	Maths Lead
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**Signed:** *B Parkinson* (Person Responsible) **Date:** 10.12.2025

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**Signed ....** *G. M. Massingham* (Chair/Governor) **Date:** 10.12.2025

# School Mission statement

*"Learning and loving together; we grow with Jesus"*

## Our Core Values

During our September 2015 INSET day we renewed both our Mission Statement and Core Values.

During the day the feedback from parents, and the views expressed by the children during sessions in the previous summer term played a prominent role in the decisions we made.



Our new Mission Statement is:

**"Learning and loving together; we grow with Jesus"**

The Core Values that provide the foundation for that Mission are:

Faithful  
Nurturing  
Respectful

Positive  
Forgiving  
Honest

Safe  
Fair

## NURTURE

The School's six nurturing principles sum up our practice and theory. They underpin the context, organisation and curriculum.

1. Children's learning is understood developmentally
2. The classroom offers a safe base
3. the importance of nurture for the development of wellbeing
4. Language as a vital means of communication
5. All behaviour is communication
6. The importance of transition in children's lives



Our Mission is represented by this design. As with the statement itself, the logo was developed by all stakeholders, with the children in particular providing the symbolic ideas of **growth – the tree, love – the hearts** and **Christ – the Cross**



## Calculation Policy Our Lady's RC Primary School



### **Background to policy**

This policy contains the key pencil and paper procedures that will be taught within our school. It has been written to ensure consistency and progression throughout the school and reflects a whole school agreement. We are grateful to the Lancashire Numeracy Team whose original ideas for a calculation policy have been adapted to meet our own.

Although the focus of the policy is on pencil and paper procedures it is important to recognise that the ability to calculate mentally lies at the heart of the Numeracy Strategy. The mental methods in the Framework for teaching mathematics will be taught systematically from Reception onwards and pupils will be given regular opportunities to develop the necessary skills. However mental calculation is not at the exclusion of written recording and should be seen as complementary to and not as separate from it. In every written method there is an element of mental processing. Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and to develop new ideas. Therefore written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

During their time at this school children will be encouraged to see mathematics as both a written and spoken language. Teachers will support and guide children through the following important stages:

- Developing the use of practical equipment, pictures and a mixture of words and symbols to represent numerical activities;
- Using standard symbols and conventions;
- Use of jottings to aid a mental strategy;
- Use of pencil and paper procedures;
- Use of a calculator.
- Use of gestures (Mastering Number)

This policy concentrates on the introduction of standard symbols, the use of the empty number line as a jotting to aid mental calculation and on the introduction of pencil and paper procedures. It is important that children do not abandon jottings and mental methods once pencil and paper procedures are introduced. Therefore children will always be encouraged to look at a calculation/problem and then decide the best method to choose – pictures, mental calculation with or without jottings, structured recording or a calculator. Our long-term aim is for children to be able to select an efficient method of their choice (whether this be mental, written or in upper Key Stage 2 using a calculator) that is appropriate for a given task.

They will do this by always asking themselves:

- 'Can I do this in my head?
- 'Can I do this in my head using drawings or jottings?'
- 'Do I need to use a pencil and paper procedure?'
- 'Do I need a calculator?'



## **Progression Towards a Written Method for Addition**

In developing a written method for addition, it is important that children understand the concept of addition, in that it is:

- Combining two or more groups to give a total or sum
- Increasing an amount

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of subtraction
- commutative i.e.  $5 + 3 = 3 + 5$
- associative i.e.  $5 + 3 + 7 = 5 + (3 + 7)$

The fact that it is commutative and associative means that calculations can be rearranged, e.g.

$4 + 13 = 17$  is the same as  $13 + 4 = 17$ .

### **YR**

#### **Early Learning Goal:**

***Using quantities and objects, children add two single-digit numbers and count on to find the answer.***

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc. This is continuously focused on with daily teaching of Mastering Number for their maths lessons which aims to:

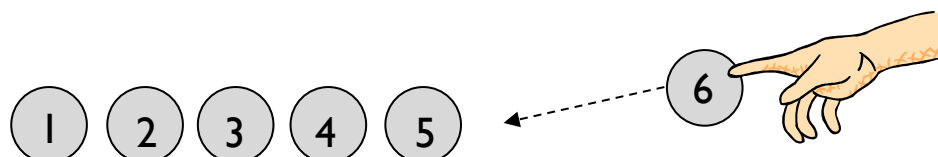
- Develop and demonstrate good number sense for pupils.
- Build firm mathematical foundations
- Develop intentional teaching strategies focused on developing fluency in calculation and number sense for all children
- Develop understanding and use of appropriate manipulatives to support your teaching of mathematical structures

### **Counting all method**

Children will begin to develop their ability to add by using practical equipment to count out the correct amount for each number in the calculation and then combine them to find the total. For example, when calculating  $4 + 2$ , they are encouraged to count out four counters and count out two counters.



To find how many altogether, touch and drag them into a line one at a time whilst counting.



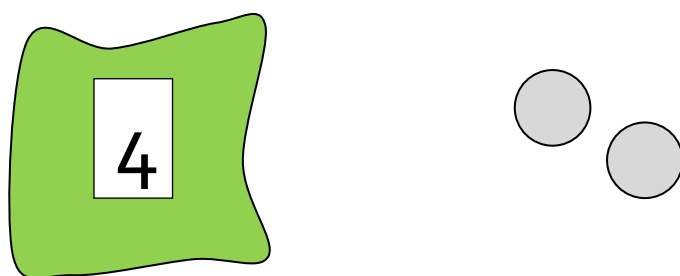
By touch counting and dragging in this way, it allows children to keep track of what they have already counted to ensure they don't count the same item twice.

### **Counting on method**

To support children in moving from a counting all strategy to one involving counting on, children should still have two groups of objects but one should be covered so that it cannot be counted. For example, when calculating  $4 + 2$ , count out the two groups of counters as before.



then cover up the larger group with a cloth.



For most children, it is beneficial to place the digit card on top of the cloth to remind the children of the number of counters underneath. They can then start their count at 4, and touch count 5 and 6 in the same way as before, rather than having to count all of the counters separately as before.

**Those who are ready** may record their own calculations.

## Y1

Year 1 and Year 2 classes use the Mastering Number materials in addition to their daily maths lesson. Children access 15 minutes daily lessons with use of Rekenrek manipulatives to support a strong number sense and number bond knowledge.

### **End of Year Objective:**

***Add one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).***

Children will continue to use practical equipment, combining groups of objects to find the total by counting all or counting on. Using their developing understanding of place value, they will move on to be able to use Base 10 equipment to make teens numbers using separate tens and units.

For example, when adding 11 and 5, they can make the 11 using a ten rod and a unit.



The units can then be combined to aid with seeing the final total, e.g.



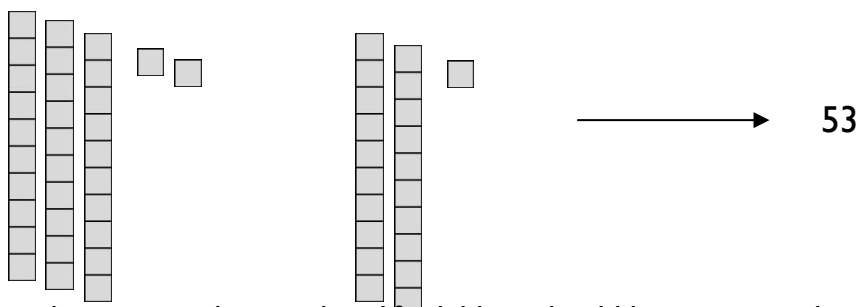
so  $11 + 5 = 16$ . If possible, they should use two different colours of base 10 equipment so that the initial amounts can still be seen.

## Y2

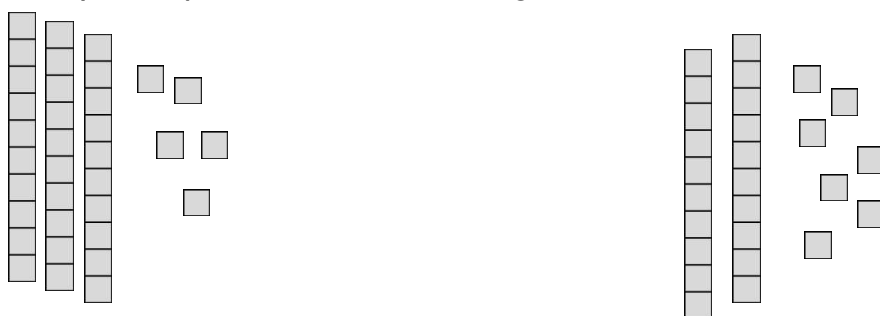
### End of Year Objective:

**Add numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers; three one-digit numbers.**

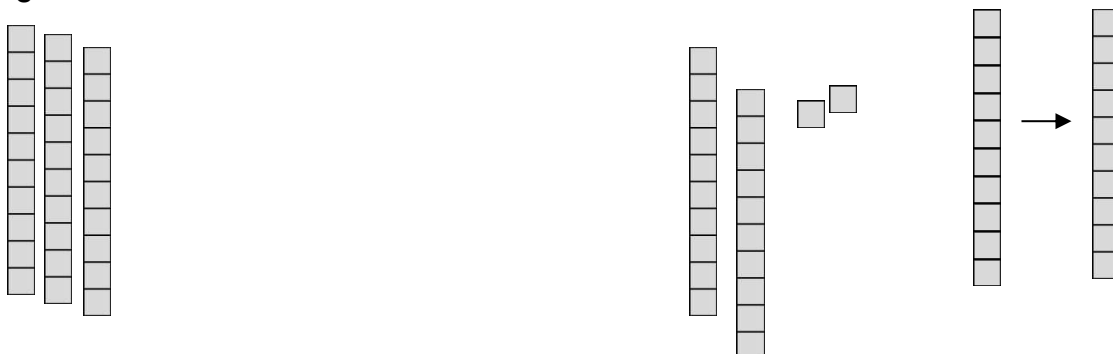
Children will continue to use the Base 10 equipment to support their calculations. For example, to calculate  $32 + 21$ , they can make the individual amounts, counting the tens first and then count on the units.



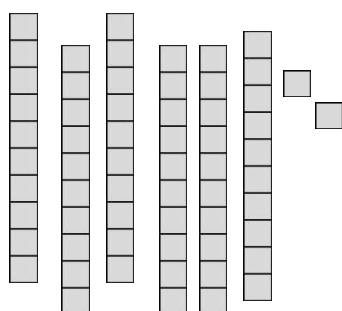
When the units total more than 10, children should be encouraged to exchange 10 units/ones for 1 ten. This is the start of children understanding 'carrying' in vertical addition. For example, when calculating  $35 + 27$ , they can represent the amounts using Base 10 as shown:



Then, identifying the fact that there are enough units/ones to exchange for a ten, they can carry out this exchange:



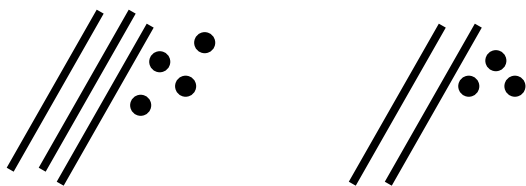
To leave:





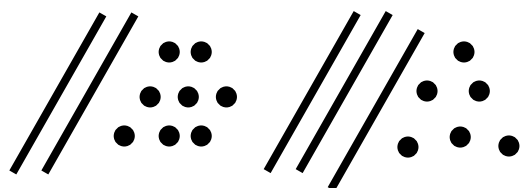
Children can also record the calculations using their own drawings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the unit blocks).

e.g.  $34 + 23 =$

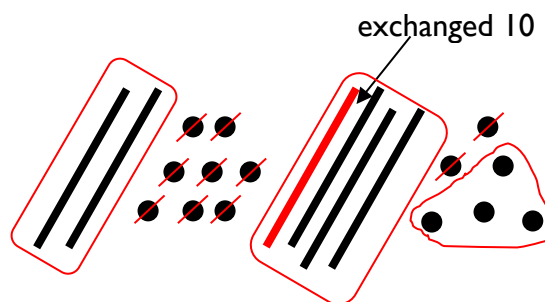


With exchange:

e.g.  $28 + 36 =$



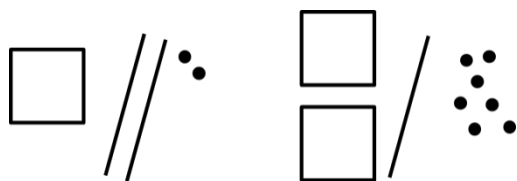
will become



so  $28 + 36 = 64$

It is important that children circle the remaining tens and units/ones after exchange to identify the amount remaining.

This method can also be used with adding three digit numbers, e.g.  $122 + 217$  using a square as the representation of 100.



### Y3

#### **End of Year Objective:**

**Add numbers with up to three digits, using formal written method of columnar addition.\***

*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4*

*It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

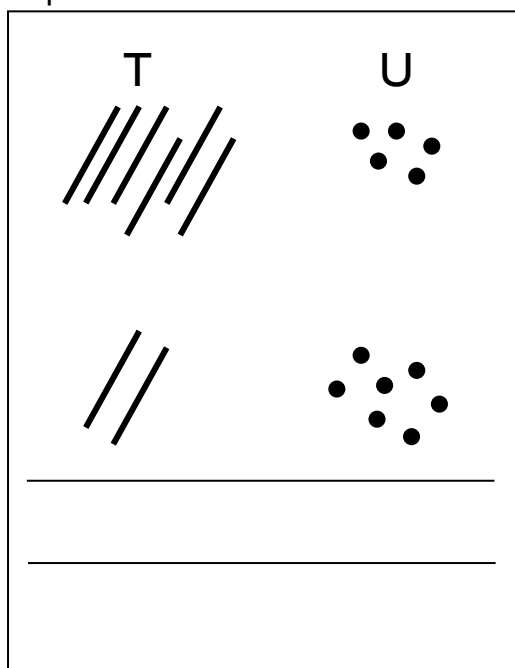
Children will build on their knowledge of using Base 10 equipment from year 2 and continue to use the idea of exchange.

Children should add the **least significant digits** first (i.e. start with the units/ones), and in an identical method to that from year 2, should identify whether there are greater than ten units which can be exchanged for one ten.

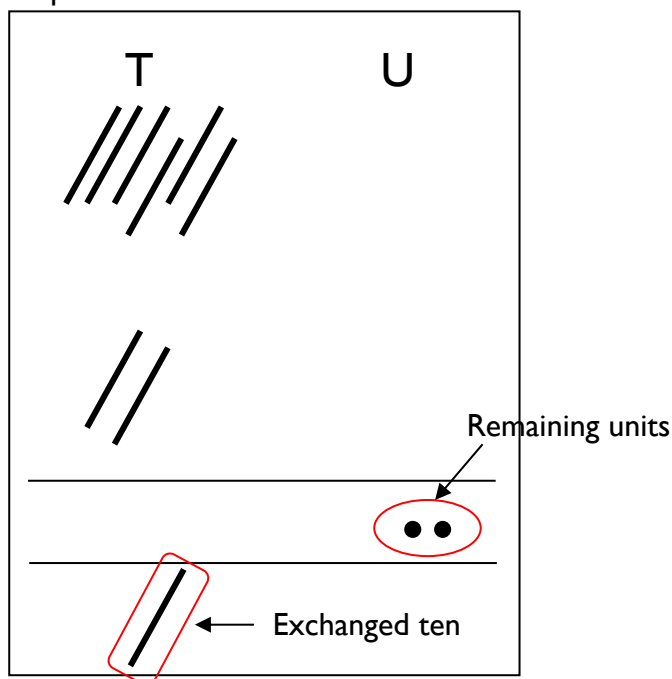
They can use a place value grid to begin to set the calculation out vertically and to support their knowledge of exchange between columns (as in Step 1 in the diagram below).

e.g.  $65 + 27$

Step 1



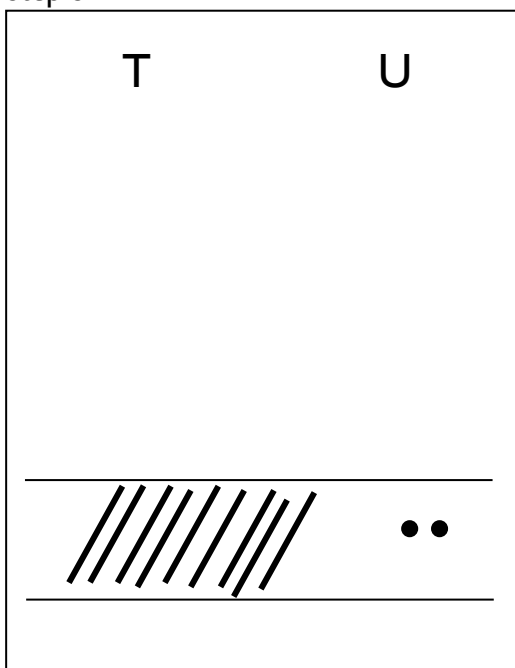
Step 2



Children would exchange ten units/ones for a ten, placing the exchanged ten below the equals sign. Any remaining units/ones that cannot be exchanged for a ten move into the equals sign as they are the units part of the answer (as in the diagram in Step 2 above).

If there are any tens that can be exchanged for a hundred, this can be done next. If not, the tens move into the equals sign as they are the tens part of the answer (as in the diagram in Step 3 below).

Step 3



Written method

Step 1	Step 2	Step 3																																						
<table><tr><td>T</td><td>U</td></tr><tr><td>6</td><td>5</td></tr><tr><td>+</td><td>2 7</td></tr><tr><td colspan="2"><hr/></td></tr><tr><td colspan="2"> </td></tr></table>	T	U	6	5	+	2 7	<hr/>				<table><tr><td>T</td><td>U</td></tr><tr><td>6</td><td>5</td></tr><tr><td>+</td><td>2 7</td></tr><tr><td colspan="2"><hr/></td></tr><tr><td colspan="2">2</td></tr><tr><td colspan="2"><hr/></td></tr><tr><td colspan="2">1</td></tr></table>	T	U	6	5	+	2 7	<hr/>		2		<hr/>		1		<table><tr><td>T</td><td>U</td></tr><tr><td>6</td><td>5</td></tr><tr><td>+</td><td>2 7</td></tr><tr><td colspan="2"><hr/></td></tr><tr><td colspan="2">9 2</td></tr><tr><td colspan="2"><hr/></td></tr><tr><td colspan="2">1</td></tr></table>	T	U	6	5	+	2 7	<hr/>		9 2		<hr/>		1	
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Children should utilise this practical method to link their understanding of exchange to how the column method is set out. Teachers should model the written method alongside this practical method initially.

This should progress to children utilising the written and practical methods alongside each other and finally, and when they are ready, to children utilising just the written method.

By the end of year 3, children should also extend this method for three digit numbers.

**Y4**

**End of Year Objective:**  
**Add numbers with up to 4 digits *and decimals with one decimal place* using the formal written method of columnar addition where appropriate.**

Children will move to year 4 using whichever method they were using as they transitioned from year 3.

**Step 1**

H

T

U

	H	T	U
	3	6	5
+	2	4	7
<hr/>			
<hr/>			

**Step 2**

H

T

U

	H	T	U
	3	6	5
+	2	4	7
<hr/>			
			2
<hr/>			
		1	

### Step 3

H	T	U
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	H	T	U
	3	6	5
+	2	4	7
		1	2
	1	1	

### Step 4

H	T	U
<hr style="border: 0; border-top: 1px solid black; margin: 10px 0;"/> <div style="display: flex; justify-content: space-between; align-items: center;"> <div style="display: flex; justify-content: space-around; width: 150px;"> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 5px;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 5px;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 5px;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 5px;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 5px;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 5px;"></div> </div> <div style="flex-grow: 1; border-left: 2px solid black; position: relative;"> <div style="position: absolute; top: -10px; left: 50%; transform: translateX(-50%);">/</div> </div> <div style="text-align: center;"> <div style="display: flex; justify-content: space-around; width: 40px;"> <div style="width: 10px; height: 10px; background: black; border-radius: 50%;"></div> <div style="width: 10px; height: 10px; background: black; border-radius: 50%;"></div> </div> </div> </div> <hr style="border: 0; border-top: 1px solid black; margin: 10px 0;"/>		

	H	T	U
	3	6	5
+	2	4	7
	6	1	2
	1	1	

By the end of year 4, children should be using the written method confidently and with understanding. They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with one decimal place, knowing that the decimal points line up under one another.*

## Y5

### End of Year Objective:

**Add whole numbers with more than 4 digits and decimals with two decimal places, including formal written methods (columnar addition).**

Children should continue to use the carrying method to solve calculations such as:

$$\begin{array}{r} 3364 \\ + 247 \\ \hline 3611 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 3121 \\ + 148 \\ \hline 3306 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 3.56 \\ + 2.47 \\ \hline 6.03 \\ \hline 1 \end{array}$$

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

## Y6

### End of Year Objective:

**Add whole numbers and decimals using formal written methods (columnar addition).**

Children should extend the carrying method and use it to add whole numbers and decimals with any number of digits.

$$\begin{array}{r} 42 \\ 6432 \\ 7863 \\ + 4681 \\ \hline 11944 \\ \hline 1121 \end{array}$$

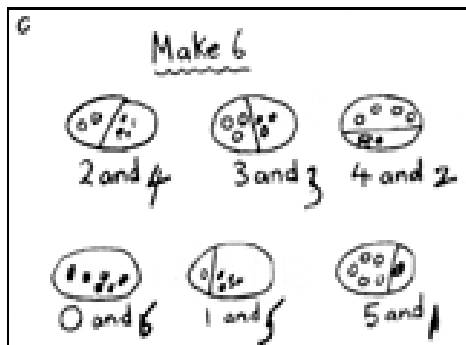
$$\begin{array}{r} 401.20 \\ 26.85 \\ + 0.71 \\ \hline 428.76 \\ \hline 1 \end{array}$$

When adding decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another.

1



Children are taught that addition is the combining of two or more amounts. They begin by counting all of the items in the groups, then move on to counting on from the largest amount. Children are encouraged to develop a mental image of the size of numbers. They learn to think about addition as combining amounts in practical, real life situations.

They begin to record addition number sentences such as  $2 + 4 = 6$  and  $8 = 3 + 5$  and  $3 + 2 + 4 = 9$

## Stage 2

## Stage 2

Children move on to using Base 10 equipment to support their developing understanding of addition.

$$11 + 5 = 16$$

11 cubes are lined up (1 ten and 1 unit/one).

5 cubes are added to the line of 11 giving a total of 16.



If possible, use two different colours of base 10 equipment so that the initial amounts can still be seen.

## Stage 3

Children continue to use the Base 10 equipment to support their calculations, including exchanging 10 units/ones for 1 ten when the total of the units/ones is 10 or more. They will record their own drawings of the Base 10 equipment, using lines for 10 rods and dots for the unit blocks.

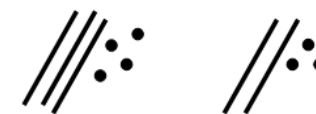
$$34 + 23 = ?$$

The units/ones are added first  $4 + 3 = 7$

The tens are added next

$$30 + 20 = 50$$

Both answers are put together  $50 + 7 = 57$



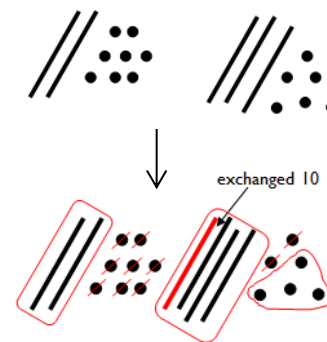
$$28 + 36 = ?$$

The units/ones are added first

$$8 + 6 = 14 \text{ with ten}$$

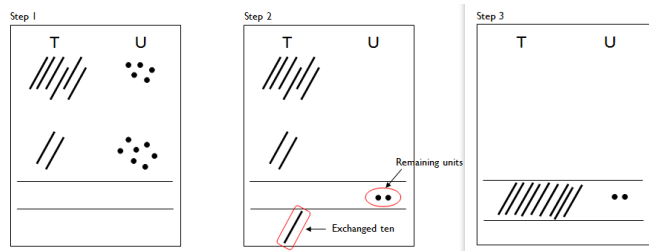
units/ones exchanged for 1 ten.

A ring is put around the units/ones not exchanged – this is the units part of the answer. The tens are then added, including the exchanged ten, to complete the sum.



## Stage 4

$$65 + 27$$



Written method

Step 1	Step 2	Step 3
T U	T U	T U
6 5	6 5	6 5
+ 2 7	+ 2 7	+ 2 7
<hr/>	<hr/>	<hr/>
	2	9 2
<hr/>	<hr/>	<hr/>
	1	1

## Stage 5

HTU			
625	367	321	£3.48
+ 48	+ 85	+ 7	+ £0.78
<hr/>	<hr/>	<hr/>	<hr/>
673	452	328	£4.26
1	11	1	1 1

This is the final stage of the method, and should be continued to be used for all written addition calculations.

The example top left would be 'said' as follows:

$5 + 8 = 13$ , put 3 down and carry the 10

$20 + 40 + 10$  that was carried over = 70 (7 written in the tens column)

$600 + 0 = 600$  (6 written in the hundreds column)

Children will be expected to use this method for adding numbers with more than 3 digits, numbers involving decimals and adding any number of amounts together.

Children should not be made to go onto the next stage if:

- 1) they are not ready.
- 2) they are not confident.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

## Progression Towards a Written Method for Subtraction

In developing a written method for subtraction, it is important that children understand the concept of subtraction, in that it is:

- Removal of an amount from a larger group (take away)
- Comparison of two amounts (difference)

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of addition
- not commutative i.e.  $5 - 3$  is not the same as  $3 - 5$
- not associative i.e.  $10 - 3 - 2$  is not the same as  $10 - (3 - 2)$

### YR

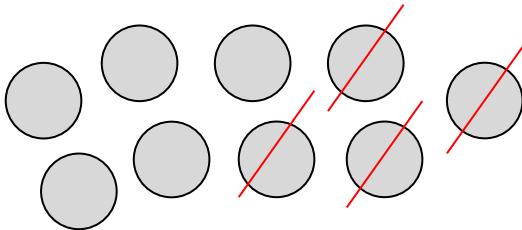
#### **Early Learning Goal:**

*Using quantities and objects, children subtract two single-digit numbers and count on or back to find the answer.*

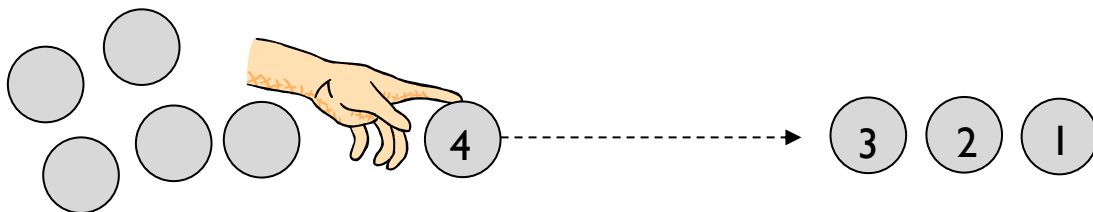
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

#### **Taking away**

Children will begin to develop their ability to subtract by using practical equipment to count out the first number and then remove or take away the second number to find the solution by counting how many are left e.g.  $9 - 4$ .



For illustration purposes, the amount being taken away are show crossed out. Children would be encouraged to physically remove these using touch counting.



By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they don't have to keep recounting. They will then touch count the amount that are left to find the answer.

**Those who are ready** may record their own calculations.

## Y1

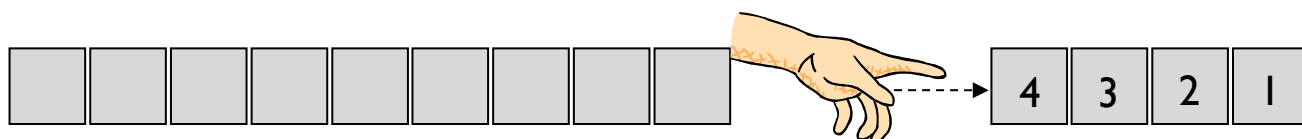
### End of Year Objective:

**Subtract one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).**

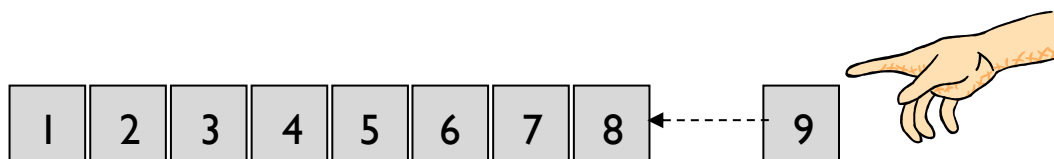
Children will continue to use practical equipment and taking away strategies. To avoid the need to exchange for subtraction at this stage, it is advisable to continue to use equipment such as counters, cubes and the units from the Base 10 equipment, but not the tens, e.g.  $13 - 4$



Touch count and remove the number to be taken away, in this case 4.



Touch count to find the number that remains.

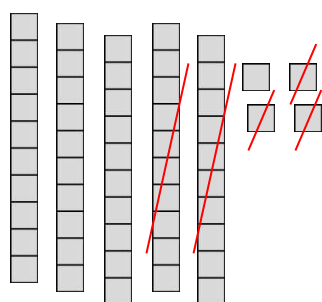


## Y2

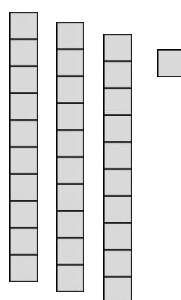
### End of Year Objective:

**Subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers.**

Children will begin to use the Base 10 equipment to support their calculations, still using a take away, or removal, method. They need to understand that the number being subtracted does not appear as an amount on its own, but rather as part of the larger amount. For example, to calculate  $54 - 23$ , children would count out 54 using the Base 10 equipment (5 tens and 4 units). They need to consider whether there are enough units/ones to remove 3, in this case there are, so they would remove 3 units and then two tens, counting up the answer of 3 tens and 1 unit to give 31.



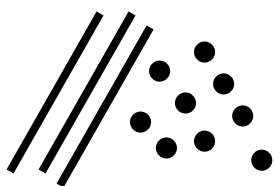
which leaves



so  $54 - 23 = 31$



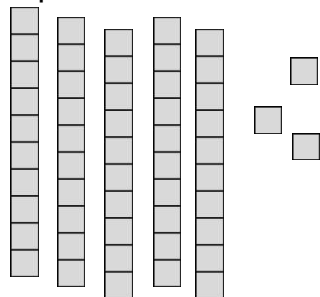
Children can also record the calculations using their own drawings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the unit blocks), e.g. to calculate  $39 - 17$  children would draw 39 as 3 tens (lines) and 4 units (dots) and would cross out 7 units and then one ten, counting up the answer of 2 tens and 2 units to give 22.



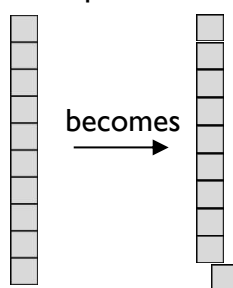
Circling the tens and units that remain will help children to identify how many remain.

When the amount of units to be subtracted is greater than the units in the original number, an exchange method is required. This relies on children's understanding of ten units being an equivalent amount to one ten. To calculate  $53 - 26$ , by using practical equipment, they would count out 53 using the tens and units, as in Step 1. They need to consider whether there are enough units/ones to remove 6. In this case there are not so they need to exchange a ten into ten ones to make sure that there are enough, as in step 2.

Step 1

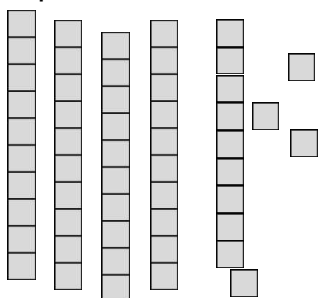


Step 2

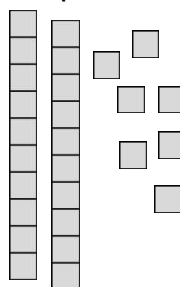


The children can now see the 53 represented as 40 and 13, still the same total, but partitioned in a different way, as in step 3 and can go on to take away the 26 from the calculation to leave 27 remaining, as in Step 4.

Step 3

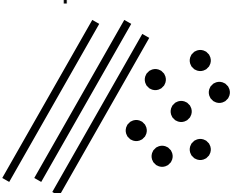


Step 4

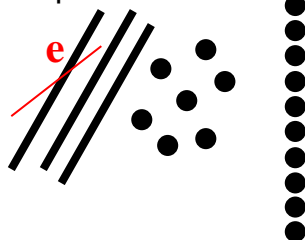


When recording their own drawings, when calculating  $37 - 19$ , children would cross out a ten and exchange for ten units. The exchanged ten is denoted with an **e** so children recognise this has not been subtracted. Drawing the units in a vertical line, as in Step 2, ensures that children create ten ones and do not get them confused with the units that were already in place.

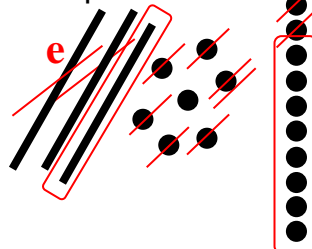
Step 1



Step 2



Step 3



Circling the tens and units that remain will help children to identify how many remain.

**End of Year Objective:**

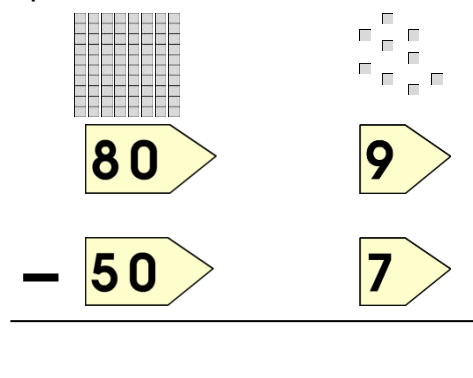
**Subtract numbers with up to three digits, using formal written method of columnar subtraction.\***

*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states “The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study.” p4*

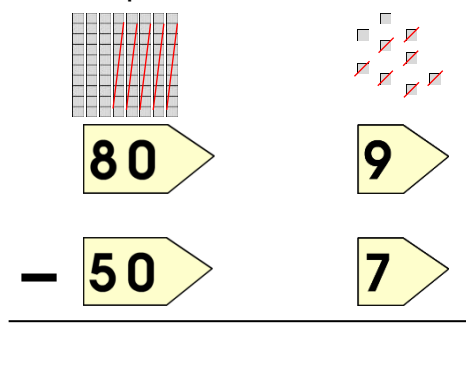
*It is more beneficial for children’s understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

Children will build on their knowledge of using Base 10 equipment from year 2 and continue to use the idea of exchange. This process should be demonstrated using arrow cards to show the partitioning and Base 10 materials to represent the first number, removing the units and tens as appropriate (as with the more informal method in year 2).

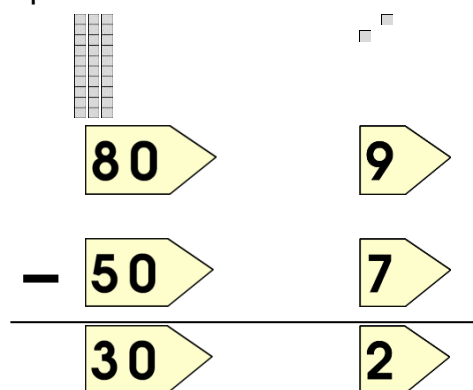
Step 1



Step 2



Step 3




*Emphasise that the second (bottom) number is being subtracted from the first (top) number rather than the lesser number from the greater.*

This will be recorded by the children as:

$$\begin{array}{r} 80 \rightarrow 9 \\ - 50 \rightarrow 7 \\ \hline 30 \rightarrow 2 \end{array} = 32$$


Children can also use jottings of the Base 10 materials (as in year 2) to support with their calculation, as in the example below.



80	→	9	
- 50	→	7	
<hr/>			
30	→	2	= 32


From this the children will begin to solve problems which involve exchange. Children need to consider whether there are enough units/ones to remove 6. In this case there are not (Step 1) so they need to exchange a ten into ten ones to make sure that there are enough, as they have been doing in the method for year 2 (Step 2). They should be able to see that the number is just partitioned in a different way, but the amount remains the same (71 = 70 + 1 = 60 + 11).

Step 1




70	1
- 40	6
<hr/>	
<hr/>	

Step 2




60	11
- 40	6
<hr/>	
<hr/>	

Step 3



60	11
- 40	6
<hr/>	
<hr/>	

Step 4



60	11
- 40	6
<hr/>	
20	5
<hr/>	

This will be recorded by the children as:

60	→	11	
- 40	→	6	
<hr/>			
20	→	5	= 25

By the end of year 3, children should also extend this method for three digit numbers.

## Y4

### End of Year Objective:

**Subtract numbers with up to 4 digits *and decimals with one decimal place* using the formal written method of columnar subtraction where appropriate.**

Children will move to year 4 using whichever method they were using as they transitioned from year 3.

#### Step 1

$$\begin{array}{r} 700 \rightarrow 50 \rightarrow 4 \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline \end{array}$$

#### Step 2 (exchanging from tens to units)

$$\begin{array}{r} 700 \rightarrow \overset{40}{\cancel{50}} \rightarrow '4 \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline \end{array}$$

#### Step 3 (exchanging from hundreds to tens)

$$\begin{array}{r} \overset{600}{\cancel{700}} \rightarrow \overset{140}{\cancel{50}} \rightarrow '4 \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline \end{array}$$

#### Step 4

$$\begin{array}{r} \overset{600}{\cancel{700}} \rightarrow \overset{140}{\cancel{50}} \rightarrow '4 \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline 400 \rightarrow 60 \rightarrow 8 = 468 \end{array}$$

This would be recorded by the children as:

$$\begin{array}{r} \overset{600}{\cancel{700}} \rightarrow \overset{140}{\cancel{50}} \rightarrow '4 \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline 400 \rightarrow 60 \rightarrow 8 = 468 \end{array}$$

When children are ready, this leads on to the compact method of decomposition:

$$\begin{array}{r} \phantom{0} 4 \overset{6}{\cancel{7}} \overset{14}{\cancel{5}} '4 \\ - \phantom{0} 3 \phantom{0} 2 \phantom{0} 8 \phantom{0} 6 \\ \hline \phantom{0} 1 \phantom{0} 4 \phantom{0} 6 \phantom{0} 8 \end{array}$$

By the end of year 4, children should be using the written method confidently and with understanding. They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with one decimal place, knowing that the decimal points line up under one another.*

## Y5

### End of Year Objective:

**Subtract whole numbers with more than 4 digits *and* decimals with two decimal places, including formal written methods (columnar subtraction).**

Children should continue to use the decomposition method to solve calculations such as:

$$\begin{array}{r} \overset{6}{7} \overset{10}{0} \overset{6}{7} \overset{12}{2} \\ - 3 \ 2 \ 2 \ 6 \\ \hline 3 \ 8 \ 4 \ 6 \end{array}$$

$$\begin{array}{r} \overset{2}{3} \overset{13}{4} \overset{12}{2} \\ - 1 \ . \ 7 \ 6 \\ \hline 1 \ . \ 6 \ 6 \end{array}$$

They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

## Y6

### End of Year Objective:

**Subtract whole numbers and decimals using formal written methods (columnar subtraction).**

Children should extend the decomposition method and use it to subtract whole numbers and decimals with any number of digits.

$$\begin{array}{r} \overset{5}{6} \overset{13}{4} \overset{13}{2} \\ - 4 \ 6 \ 8 \ 1 \\ \hline 1 \ 7 \ 5 \ 1 \end{array}$$

$$\begin{array}{r} \overset{3}{4} \overset{11}{7} \overset{11}{2} \\ - 3 \ 8 \ 2 \ . \ 4 \ 5 \\ \hline \end{array}$$

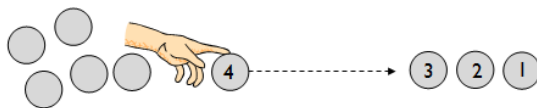
When subtracting decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another.

### Stage 1

Children will subtract two numbers by taking one away from the other and counting how many are left.



Children are encouraged to develop a mental image of the size of numbers. They learn to think about subtraction as 'take away' in practical, real life situations.

They begin to record subtraction number sentences such as  $8 - 5 = 3$



### Stage 4A

$$\begin{array}{r} 89 \\ - 57 \\ \hline \end{array} = \begin{array}{r} 80 \rightarrow 9 \\ 50 \rightarrow 7 \\ 30 \rightarrow 2 = 32 \end{array}$$

The calculation should be read as subtract 7 from 9 or 9 subtract 7.

Children move from using the Base 10 method to expanded vertical method, using base 10 notation and arrow cards. Children learn to subtract the least significant digits first (start with the numbers on the right and work from right to left). The answer to each individual subtraction is written underneath before these answers are recombined.

### Stage 2

Children move on to using Base 10 equipment alongside a number track to support their developing understanding of subtraction.

$$13 - 4 = ?$$

13 cubes are lined up.

4 cubes are removed from the end of the line leaving 9 left. It is important that children keep track of how many have been removed.



Touch count and remove the number to be taken away.



Touch count to find the number that remains.



### Stage 4B

This stage involves exchange.

It is clear that there are not enough units to subtract 6 from 1, so one of the tens from the 70 is exchanged for 10 ones.

The initial number 71 is rearranged as 60 and 11 to make the calculation easier.

This would be recorded by the children as:

$$\begin{array}{r} 70 \\ - 40 \\ \hline 30 \end{array} \quad \begin{array}{r} 1 \\ \rightarrow 11 \\ - 6 \\ \hline 5 \end{array} = 25$$

### Stage 3

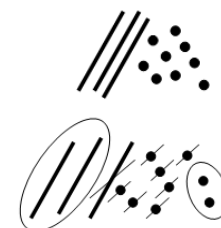
Children continue to use the Base 10 equipment to support their calculations. They will record their own drawings of the Base 10 equipment, using lines for 10 rods and dots for the unit blocks.

$$39 - 17 = ?$$

39 is drawn

17 is crossed out

A ring is drawn around what is left to give the answer giving 22



Step 1

$$37 - 19 = ?$$

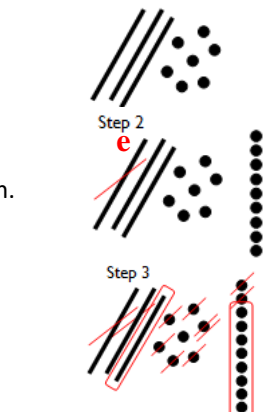
37 is drawn

9 units cannot be crossed out, so a ten is crossed out and exchanged for 10 ones which are in a line.

e is written next to the exchanged ten.

19 is crossed out

A ring is drawn around what is left to give the answer 18



Step 2

Step 3

### Stage 5

This final stage is the compact method of decomposition. The example shows how the same calculation would be carried out using the previous method and the final method.

#### Stage 4B

$$\begin{array}{r} 600 \\ - 700 \\ \hline 600 \end{array} \quad \begin{array}{r} 140 \\ \rightarrow 50 \rightarrow 14 \\ 80 \rightarrow 6 \\ 600 \rightarrow 60 \rightarrow 8 = 668 \end{array}$$

becomes

#### Stage 5

$$\begin{array}{r} 6141 \\ - 764 \\ \hline 668 \end{array}$$

This is the final stage of the process and will continue to be used with larger numbers and numbers involving decimals.

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## Progression Towards a Written Method for Multiplication

In developing a written method for multiplication, it is important that children understand the concept of multiplication, in that it is:

- repeated addition

They should also be familiar with the fact that it can be represented as an array

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of division
- commutative i.e.  $5 \times 3$  is the same as  $3 \times 5$
- associative i.e.  $2 \times 3 \times 5$  is the same as  $2 \times (3 \times 5)$

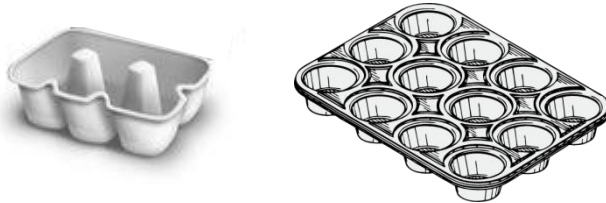
### YR

#### **Early Learning Goal:**

***Children solve problems, including doubling.***

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays.



They may develop ways of recording calculations using pictures, etc.



A child's jotting showing the fingers on each hand as a double.



A child's jotting showing double three as three cookies on each plate.

#### **End of Year Objective:**

**Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.**

In year one, children will continue to solve multiplication problems using practical equipment and jottings. They may use the equipment to make groups of objects. Children should see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc. and use this in their learning, answering questions such as 'How many eggs would we need to fill the egg box? How do you know?'



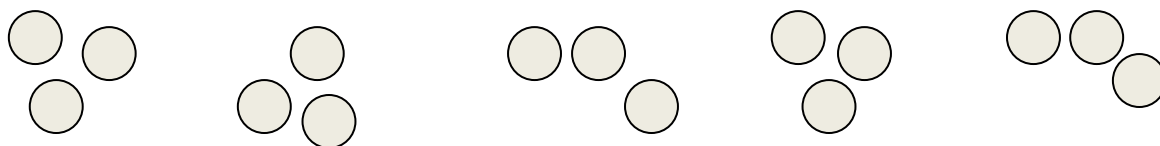
## Y2

### End of Year Objective:

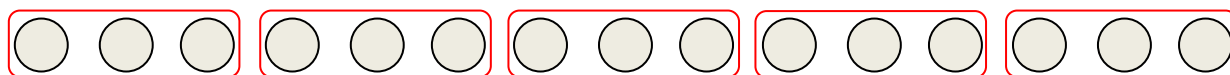
Calculate mathematical statements for multiplication (*using repeated addition*) and write them using the multiplication (x) and equals (=) signs.

Children should understand and be able to calculate multiplication as repeated addition, supported by the use of practical apparatus such as counters or cubes. e.g.

$5 \times 3$  can be shown as five groups of three with counters, either grouped in a random pattern, as below:

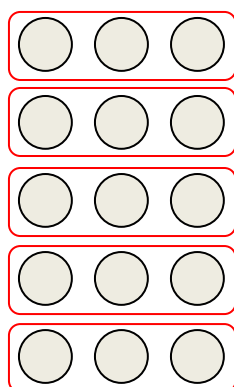


or in a more ordered pattern, with the groups of three indicated by the border outline:

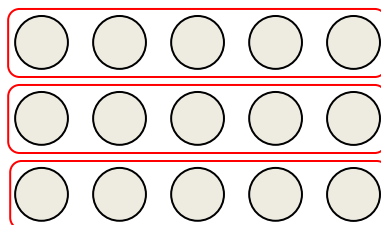


Children should then develop this knowledge to show how multiplication calculations can be represented by an array, (this knowledge will support with the development of the grid method in the future). Again, children should be encouraged to use practical apparatus and jottings to support their understanding, e.g.

$5 \times 3^*$  can be represented as an array in two forms (as it has commutativity):



$$3 + 3 + 3 + 3 + 3 = 15$$



$$5 + 5 + 5 = 15$$

\*For mathematical accuracy  $5 \times 3$  is represented by the second example above, rather than the first as it is five, three times. However, because we use terms such as 'groups of' or 'lots of', children are more familiar with the initial notation. Once children understand the commutative order of multiplication the order is irrelevant).

**End of Year Objective:**

**Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods.\***

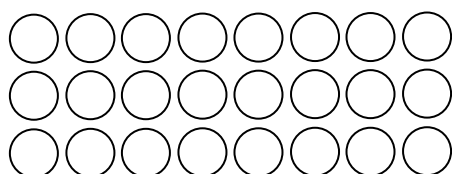
*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states “The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study.” p4*

*It is more beneficial for children’s understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

Initially, children will continue to use arrays where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

$$3 \times 8$$

They may show this using practical equipment:



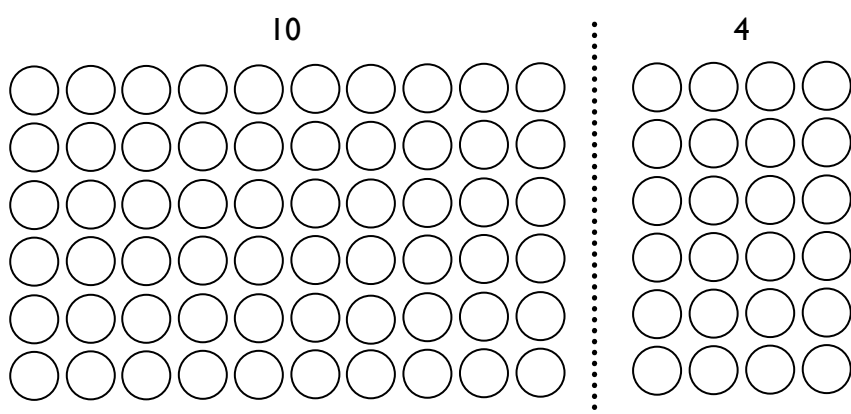
$$3 \times 8 = 8 + 8 + 8 = 24$$

or by jottings using squared paper:

	x	x	x	x	x	x	x	x	
	x	x	x	x	x	x	x	x	
	x	x	x	x	x	x	x	x	

$$3 \times 8 = 8 + 8 + 8 = 24$$

As they progress to multiplying a two-digit number by a single digit number, children should use their knowledge of partitioning two digit numbers into tens and units/ones to help them. For example, when calculating  $14 \times 6$ , children should set out the array, then partition the array so that one array has ten columns and the other four.



Partitioning in this way, allows children to identify that the first array shows  $10 \times 6$  and the second array shows  $4 \times 6$ . These can then be added to calculate the answer:

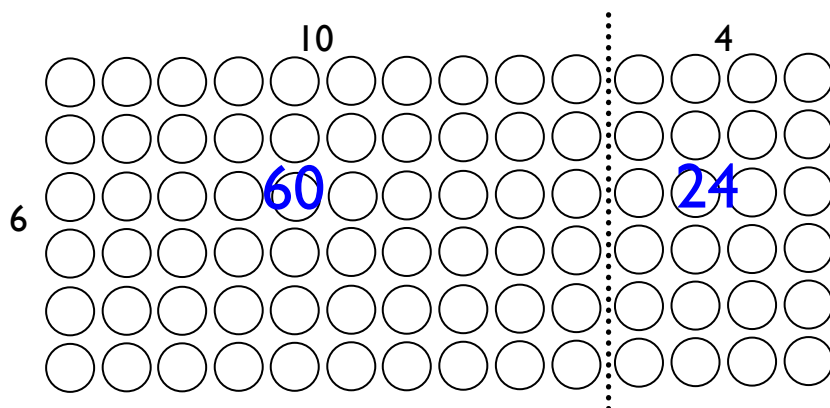
$$(6 \times 10) + (6 \times 4)$$

$$= 60 + 24$$

$$= 84$$

NB There is no requirement for children to record in this way, but it could be used as a jotting to support development if needed.

This method is the precursor step to the grid method. Using a two-digit by single digit array, they can partition as above, identifying the number of rows and the number of columns each side of the partition line.



By placing a box around the array, as in the example below, and by removing the array, the grid method can be seen.

x	10	4
6	60	24

It is really important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording.

From this, children can use the grid method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20. Children should be encouraged to set out their addition in a column at the side to ensure the place value is maintained. When children are working with numbers where they can confidently and correctly calculate the addition mentally, they may do so.

$$13 \times 8$$

x	10	3
8	80	24

$$\begin{array}{r}
 80 \\
 + 24 \\
 \hline
 104
 \end{array}$$

When children are ready, they can then progress to using this method with other two-digit numbers.

37 x 6

x	30	7
6	180	42

+ 180  
42  
222

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

## Y4

### End of Year Objective:

**Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.**

Children will move to year 4 using whichever method they were using as they transitioned from year 3. They will further develop their knowledge of the grid method to multiply any two-digit by any single-digit number, e.g.

$79 \times 8$

x	70	9
8	560	72

$$\begin{array}{r} 560 \\ + 72 \\ \hline 632 \end{array}$$

To support the grid method, children should develop their understanding of place value and facts that are linked to their knowledge of tables. For example, in the calculation above, children should use their knowledge that  $7 \times 8 = 56$  to know that  $70 \times 8 = 560$ .

By the end of the year, they will extend their use of the grid method to be able to multiply three-digit numbers by a single digit number, e.g.

$346 \times 8$

x	300	40	6
8	2400	320	48

$$\begin{array}{r} 2400 \\ + 320 \\ + 48 \\ \hline 2768 \end{array}$$

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Children will also be shown the short multiplication method when ready.

## Y5

### End of Year Objective:

**Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.**

Children should continue to use the grid method and extend it to multiplying numbers with up to four digits by a single digit number, e.g.

$$4346 \times 8$$

x	4 000	300	40	6
8	32 000	2400	320	48

$$\begin{array}{r} 32000 \\ + 2400 \\ + 320 \\ + 48 \\ \hline 34768 \end{array}$$

and numbers with up to four digits by a two-digit number, e.g.

$$2693 \times 24$$

x	2000	600	90	3
20	40000	12000	1800	60
4	8000	2400	360	12

$$\begin{array}{r} 40000 \\ + 8000 \\ + 12000 \\ + 2400 \\ + 1800 \\ + 360 \\ + 60 \\ + 12 \\ \hline 64632 \\ \text{I I I} \end{array}$$

The long list of numbers in the addition part can be used to check that all of the answers from the grid have been included, however, when children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they should be encouraged to do so.

For example,

x	2000	600	90	3	
20	40000	12000	1800	60	= 53 860
4	8000	2400	360	12	= 10 772 +
					<hr/> 64 632 <hr/> <div>I I</div>

Adding across mentally, leads children to finding the separate answers to:

$$2\,693 \times 20$$

$$2\,693 \times 4$$

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

During Year 5, the transition from the grid method into the formal vertical method for multiplication should take place. The traditional vertical compact method of written multiplication is a highly efficient way to calculate, but it has a very condensed form and needs to be introduced carefully.

It is most effective to begin with the grid method, moving to an expanded vertical layout, before introducing the compact form. This allows children to see, and understand, how the processes relate to each other and where the individual multiplication answers come from e.g.

$$368 \times 6$$

x	300	60	8	
6	1 800	360	48	
				1800 + 360 + 48 ----- 2208

Th	H	T	U
	3	6	8
x			6
		4	8
	3	6	0
+	1	8	0
	2	2	0
			8

Th	H	T	U
	3	6	8
x			6
		4	8
	3	6	0
+	1	8	0
	2	2	0
			8

becomes

Th	H	T	U
	3	6	8
x			6
		4	8
	3	6	0
+	1	8	0
	2	2	0
			8

The place value columns are labelled to ensure children understand the size of the partitioned digits in the original number(s) and in the answer.

It is vital that the teacher models the correct language when explaining the process of the compact method.

The example shown should be explained as:

*“Starting with the least significant digit... 8 multiplied by 6 is 48, put 8 in the units and carry 4 tens (40).*

*6 tens multiplied by 6 are 36 tens. Add the 4 tens carried over to give 40 tens (which is the same as 4 hundreds and 0 tens).*

*Put 0 in the tens place of the answer and carry 4 hundreds.*

*3 hundreds multiplied by 6 are 18 hundreds. Add the 4 hundreds carried over to give 22 hundreds (which is the same as 2 thousands and 2 hundreds). Write 2 in the hundreds place of the answer and 2 in the thousands place of the answer.”*

Children should recognise that the answer is close to an estimated answer of  $400 \times 6 = 2\,400$

Long multiplication could also be introduced by comparing the grid method with the compact vertical method.

Mentally totalling each row of answers is an important step in children making the link between the grid method and the compact method.

x	600	90	3	
20	12000	1800	60	= 13 860
4	2400	360	12	= 2 772 +
				16 632

Children should only be expected to move towards this next method if they have a secure understanding of place value. It is difficult to explain the compact method without a deep understanding of place value.

The example shown should be explained as:

*“Starting with the least significant digit... 3 multiplied by 4 is 12; put 2 in the units and carry 1 ten (10).*

*9 tens multiplied by 4 are 36 tens. Add the 1 ten carried over to give 37 tens (which is the same as 3 hundreds and 7 tens). Put 7 in the tens place of the answer and carry 3 hundreds.*

*6 hundreds multiplied by 4 are 24 hundreds. Add the 3 hundreds carried over to give 27 hundreds (which is the same as 2 thousands and 7 hundreds). Write 7 in the hundreds place of the answer and 2 in the thousands place of the answer. We have now found the answer to  $693 \times 4$ . Step 1 is complete so to avoid confusion later, we will cross out the carried digits 3 and 1.”*

Notice this answer can clearly be seen in the grid method example.

Step 1

$$\begin{array}{r}
 \text{TTh Th H T U} \\
 693 \\
 \times 24 \\
 \hline
 2772 \quad (693 \times 4) \\
 \begin{array}{c} \text{3} \quad \text{1} \end{array}
 \end{array}$$

Step 2

$$\begin{array}{r}
 \text{TTh Th H T U} \\
 693 \\
 \times 24 \\
 \hline
 2772 \quad (693 \times 4) \\
 + 13860 \quad (693 \times 20) \\
 \hline
 \begin{array}{c} \text{1} \quad \text{3} \quad \text{8} \quad \text{6} \quad \text{0} \end{array}
 \end{array}$$

Now we are multiplying 693 by 20. Starting with the least significant digit of the top number... 3 multiplied by 20 is 60. Write this answer in.

90 multiplied by 20 is 1 800. There are no units and no tens in this answer, so write 8 in the hundreds place and carry 1 in the thousands.

600 multiplied by 20 is 12 000. Add the 1 (thousand) that was carried to give 13 000. There are no units, no tens and no hundreds in this answer, so write 3 in the thousands place and 1 in the ten thousands place.

Step 3

$$\begin{array}{r}
 \text{TTh Th H T U} \\
 693 \\
 \times 24 \\
 \hline
 2772 \quad (693 \times 4) \\
 + 13860 \quad (693 \times 20) \\
 \hline
 16632
 \end{array}$$

The final step is to total both answers using efficient columnar addition.

When using the compact method for long multiplication, all carried digits should be placed below the line of that answer e.g.  $3 \times 4$  is 12, so the 2 is written in the units column and the 10 is carried as a small 1 in the tens column. This carrying below the answer is in line with the written addition policy in which carried digits are always written below the answer/line.



## Y6

### End of Year Objective:

**Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.**

**Multiply one-digit numbers with up to two decimal places by whole numbers.**

By the end of year 6, children should be able to use the grid method and the compact method to multiply any number by a two-digit number. They could also develop the method to be able to multiply decimal numbers with up to two decimal places, but having been introduced to expanded and compact vertical methods in Year 5, it may be appropriate to use the expanded vertical method when introducing multiplication involving decimals.

$$4.92 \times 3$$

$$\begin{array}{r} \text{T U . t h} \\ 4 . 9 2 \\ \times \quad 3 \\ \hline 0 . 0 6 \quad (0.02 \times 3) \\ 2 . 7 \quad (0.9 \times 3) \\ + 1 2 \quad (4 \times 3) \\ \hline 1 4 . 7 6 \end{array}$$

becomes

$$\begin{array}{r} \text{T U . t h} \\ 4 . 9 2 \\ \times \quad 3 \\ \hline 1 4 . 7 6 \\ 2 \end{array}$$

Children should also be using this method to solve problems and multiply numbers, including those with decimals, in the context of money or measures, e.g. to calculate the cost of 7 items at £8.63 each, or the total length of six pieces of ribbon of 2.28m each.

### Stage 1

Children are encouraged to develop a mental image of the size of numbers. They learn to think about equal groups or sets of objects in practical, real life situations. They begin to record these situations using pictures.



A child's jotting showing fingers on each hand as a double.



A child's jotting showing double three as three cookies on each plate.

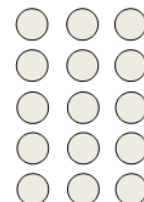
### Stage 2

Children understand that multiplication is repeated addition and that can be done by counting in equal steps/groups.



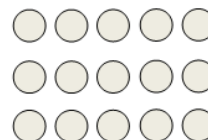
or

Children can then be introduced to the image of a rectangular array, initially through real items such as egg boxes, baking trays, ice cube trays, wrapping paper etc. and using these to show that counting up in equal groups can be a quicker way of finding a total.



$$3 + 3 + 3 + 3 + 3 = 15$$

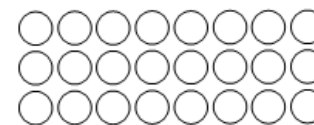
Children also understand that  $3 \times 5$  is the same as  $5 \times 3$



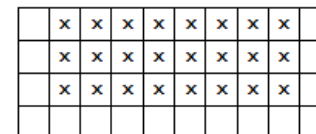
$$5 + 5 + 5 = 15$$

### Stage 3

Children continue to use arrays and create their own to represent multiplication calculations



$$3 \times 8 = 8 + 8 + 8 = 24$$



$$3 \times 8 = 8 + 8 + 8 = 24$$

### Stage 4

Children will continue to use arrays to lead into the grid method of multiplication.

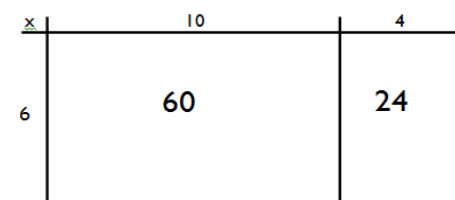
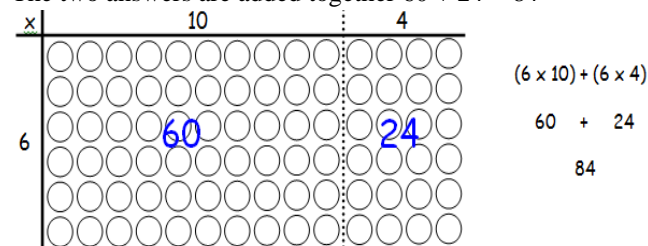
$14 \times 6$

The 14 is partitioned (split) into 10 and 4.

The answer to  $6 \times 10$  is found = 60

The answer to  $6 \times 4$  is found = 24

The two answers are added together  $60 + 24 = 84$



### Stage 5

In this stage, the array is removed and children use the grid method. This is an important step in retaining children's understanding of multiplication.

$$23 \times 8$$

$$\begin{array}{r} \times \quad 20 \quad 3 \\ 8 \quad \boxed{160} \quad \boxed{24} \end{array}$$

$$\begin{array}{r} 160 \\ + 24 \\ \hline 184 \end{array}$$

$$346 \times 9$$

$$\begin{array}{r} \times \quad 300 \quad 40 \quad 6 \\ 9 \quad \boxed{2700} \quad \boxed{360} \quad \boxed{54} \end{array}$$

$$\begin{array}{r} 2700 \\ + 360 \\ + 54 \\ \hline 3114 \\ \hline \end{array}$$

The grid method can be used for multiplying any numbers, including long multiplication and multiplication involving decimals.

$$4.92 \times 38$$

$$\begin{array}{r} \times \quad 70 \quad 2 \\ 30 \quad \boxed{2100} \quad \boxed{60} \\ 8 \quad \boxed{560} \quad \boxed{16} \end{array}$$

$$\begin{array}{r} 2100 \\ + 560 \\ + 60 \\ + 16 \\ \hline 2736 \\ \hline \end{array}$$



## Progression Towards a Written Method for Division

In developing a written method for division, it is important that children understand the concept of division, in that it is:

- repeated subtraction

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of multiplication
- not commutative i.e.  $15 \div 3$  is not the same as  $3 \div 15$
- not associative i.e.  $30 \div (5 \div 2)$  is not the same as  $(30 \div 5) \div 2$

### YR

#### **Early Learning Goal:**

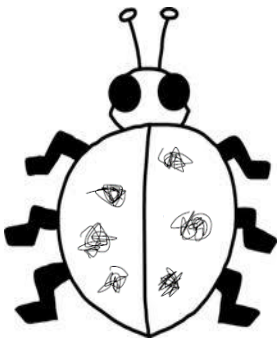
***Children solve problems, including halving and sharing.***

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins which are arrays.



They may develop ways of recording calculations using pictures, etc.



A child's jotting showing halving six spots between two sides of a ladybird.



A child's jotting showing how they shared the apples at snack time between two groups.

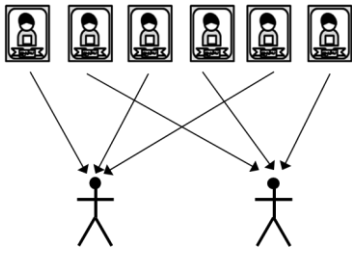


### Y1

#### **End of Year Objective:**

**Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.**

In year one, children will continue to solve division problems using practical equipment and jottings. They should use the equipment to share objects and separate them into groups, answering questions such as 'If we share these six apples between the three of you, how many will you each have? How do you know?' or 'If six football stickers are shared between two people, how many do they each get?' They may solve both of these types of question by using a 'one for you, one for me' strategy until all of the objects have been given out.



Children should be introduced to the concept of simple remainders in their calculations at this practical stage, being able to identify that the groups are not equal and should refer to the remainder as ‘... left over’.

## Y2

### End of Year Objective:

**Calculate mathematical statements for division within the multiplication tables and write them using the division ( $\div$ ) and equals (=) signs.**

Children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation, e.g.

$$12 \div 3 =$$

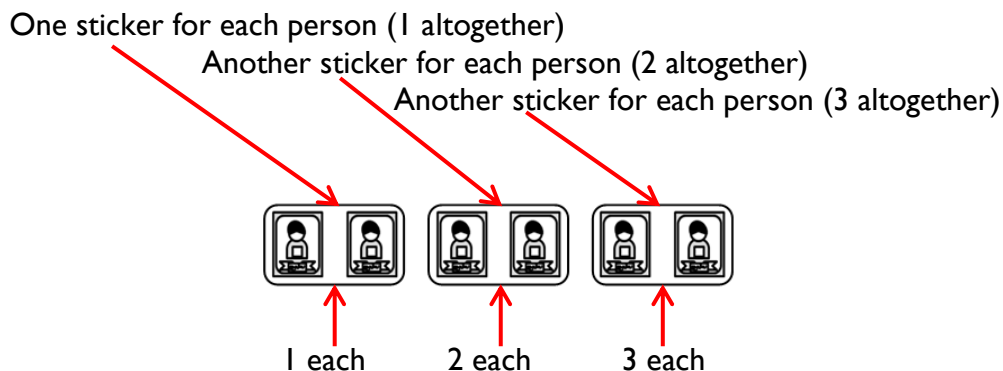


Children need to understand that this calculation reads as 'How many groups of 3 are there in 12?'

The link between sharing and grouping can be modelled in the following way:

To solve the problem ‘If six football stickers are shared between two people, how many do they each get?’

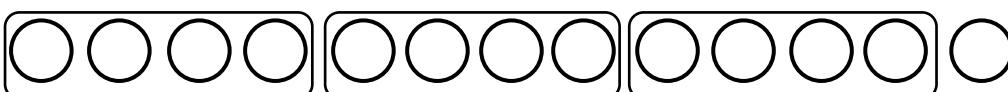
Place the football stickers in a bag or box and ask the children how many stickers would need to be taken out of the box to give each person one sticker each (i.e. 2) and exemplify this by putting the cards in groups of 2 until all cards have been removed from the bag.



Or:

Children should also continue to develop their knowledge of division with remainders, e.g.

$$13 \div 4 =$$



$$13 \div 4 = 3 \text{ remainder } 1$$

Children need to be able to make decisions about what to do with remainders after division and round up or down accordingly. In the calculation  $13 \div 4$ , the answer is 3 remainder 1, but whether the answer should be rounded up to 4 or rounded down to 3 depends on the context, as in the examples below:

I have £13. Books are £4 each. How many can I buy?

Answer: 3 (the remaining £1 is not enough to buy another book)

Apples are packed into boxes of 4. There are 13 apples. How many boxes are needed?

Answer: 4 (the remaining 1 apple still need to be placed into a box)

**End of Year Objective:**

**Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, progressing to formal written methods.\***

*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states “The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study.” p4*

*It is more beneficial for children’s understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

Initially, children will continue to use division by grouping (including those with remainders), where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

$$43 \div 8 =$$



$$43 \div 8 = 5 \text{ remainder } 3$$

In preparation for developing the ‘chunking’ method of division, children should first use the repeated subtraction on a vertical number line alongside the continued use of practical equipment. There are two stages to this:

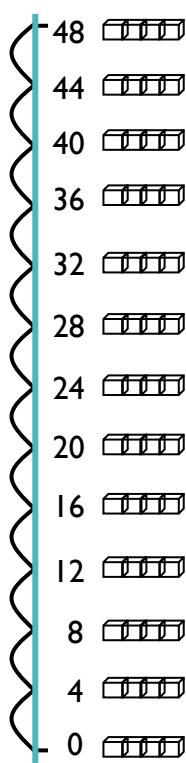
Stage 1 – repeatedly subtracting individual groups of the divisor

Stage 2 – subtracting multiples of the divisor (initially 10 groups and individual groups, then 10 groups and other multiples in line with tables knowledge)

After each group has been subtracted, children should consider how many are left to enable them to identify the amount remaining on the number line.

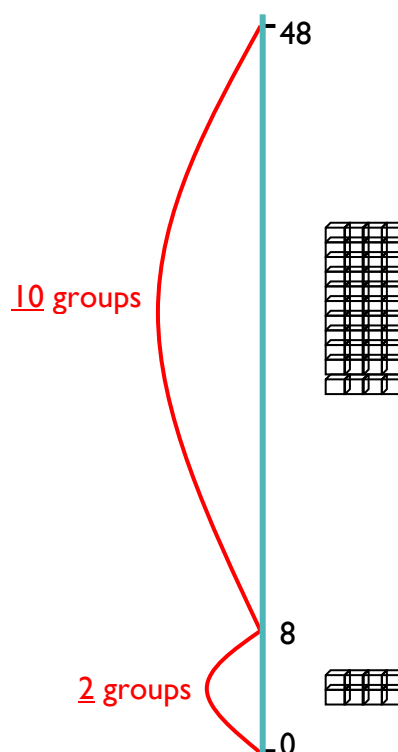
Stage 1

$$48 \div 4 = 12 \text{ (groups of 4)}$$



Stage 2

$$48 \div 4 = 10 \text{ (groups of 4)} + 2 \text{ (groups of 4)} \\ = 12 \text{ (groups of 4)}$$



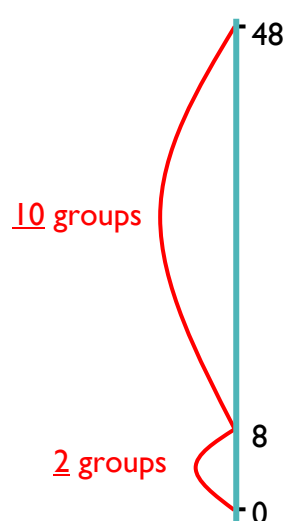
Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

## Y4

### End of Year Objective:

**Divide numbers up to 3 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.**

Children will continue to develop their use of grouping (repeated subtraction) to be able to subtract multiples of the divisor, moving on to the use of the 'chunking' method.



$$\begin{array}{r}
 12 \\
 4 \overline{) 48} \\
 \underline{- 40} \quad 10x \\
 8 \\
 \underline{- 8} \quad 2x \\
 0
 \end{array}$$

Answer: 12

Children should write their answer above the calculation to make it easy for them and the teacher to distinguish.

The number line method used in year 3 can be linked to the chunking method to enable children to make links in their understanding.

When developing their understanding of 'chunking', children should utilise a 'key facts' box, as shown below. This enables an efficient recall of tables facts and will help them in identifying the largest group they can subtract in one chunk. Any remainders should be shown as integers, e.g.

$$73 \div 3$$

$$\begin{array}{r}
 24r1 \\
 3 \overline{) 73} \\
 \underline{- 30} \quad 10x \\
 43 \\
 \underline{- 30} \quad 10x \\
 13 \\
 \underline{- 6} \quad 2x \\
 7 \\
 \underline{- 6} \quad 2x \\
 1
 \end{array}$$

Key facts box

1x	3
2x	6
5x	15
10x	30



By the end of year 4, children should be able to use the chunking method to divide a three digit number by a single digit number. To make this method more efficient, the key facts in the menu box should be extended to include 4x and 20x, e.g.

$$196 \div 6$$

$$\begin{array}{r} 32\text{r}4 \\ 3 \overline{) 196} \\ - 120 \quad 20\text{x} \\ \hline 76 \\ - 60 \quad 10\text{x} \\ \hline 16 \\ - 12 \quad 2\text{x} \\ \hline 4 \end{array}$$

Key facts box

1x	6
2x	12
4x	24
5x	30
10x	60
20x	120

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

## Y5

### End of Year Objective:

**Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.**

Children may continue to use the key facts box for as long as they find it useful. Using their knowledge of linked tables facts, children should be encouraged to use higher multiples of the divisor. **During Year 5, children should be encouraged to be efficient when using the chunking method and not have any subtraction steps that repeat a previous step. For example, when performing  $347 \div 8$  an initial subtraction of 160 ( $20 \times 8$ ) and a further subtraction of 160 ( $20 \times 8$ ) should be changed to a single subtraction of 320 ( $40 \times 8$ ). Also, any remainders should be shown as integers, e.g.**

$$523 \div 8$$

$$\begin{array}{r} 65\text{r}3 \\ 8 \overline{) 523} \\ - 320 \quad 40\text{x} \\ \hline 203 \\ - 160 \quad 20\text{x} \\ \hline 43 \\ - 40 \quad 5\text{x} \\ \hline 3 \end{array}$$

By the end of year 5, children should be able to use the chunking method to divide a four digit number by a single digit number. If children still need to use the key facts box, it can be extended to include 100x.

$$2458 \div 7$$

$$\begin{array}{r} 351\text{r}1 \\ 7 \overline{) 2458} \\ - 2100 \quad 300\text{x} \\ \hline 358 \\ - 350 \quad 50\text{x} \\ \hline 8 \\ - 7 \quad 1\text{x} \\ \hline 1 \end{array}$$

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

**Y6**

**End of Year Objective:**

**Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.**

**Use written division methods in cases where the answer has up to two decimal places.**

To develop the chunking method further, it should be extended to include dividing a four-digit number by a two-digit number, e.g.

$$6367 \div 28$$

$$\begin{array}{r} 227r11 \\ 28 \overline{) 6367} \\ \underline{- 5600} \phantom{00} 200x \\ 767 \phantom{00} 20x \\ \underline{- 560} \phantom{00} 5x \\ 207 \phantom{00} 2x \\ \underline{- 140} \phantom{00} \\ 67 \\ \underline{- 56} \\ 11 \end{array}$$

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

In addition, children should also be able to use the chunking method and solve calculations interpreting the remainder as a decimal up to two decimal places.

This should first be demonstrated using a simple calculation such as  $13 \div 4$  to show the remainder initially as a fraction.



Using practical equipment, children can see that for  $13 \div 4$ , the answer is 3 remainder 1, or put another way, there are three whole groups and a remainder of 1. This remainder is one part towards a full group of 4, so is  $\frac{1}{4}$ . To show the remainder as a fraction, it becomes the numerator where the denominator is the divisor (the number that you are dividing by in the calculation).

$$3574 \div 8$$

$$\begin{array}{r} 446r6 \\ 8 \overline{) 3574} \\ \underline{- 3200} \phantom{00} 400x \\ 374 \phantom{00} 40x \\ \underline{- 320} \phantom{00} 6x \\ 54 \\ \underline{- 48} \\ 6 \end{array}$$

$$\frac{6}{8} \leftarrow \frac{\text{remainder}}{\text{divisor}}$$

So  $3574 \div 8$  is  $446\frac{6}{8}$   
(when the remainder is shown as a fraction)

To show the remainder as a decimal relies upon children's knowledge of decimal fraction equivalents. For decimals with no more than 2 decimal places, they should be able to identify:

Half:  $\frac{1}{2} = 0.5$

Quarters:  $\frac{1}{4} = 0.25$ ,  $\frac{3}{4} = 0.75$

Fifths:  $\frac{1}{5} = 0.2$ ,  $\frac{2}{5} = 0.4$ ,  $\frac{3}{5} = 0.6$ ,  $\frac{4}{5} = 0.8$

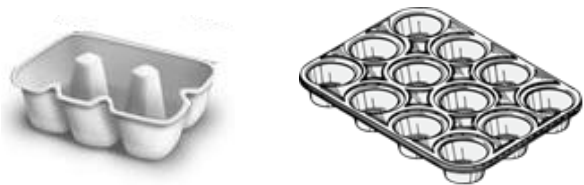
Tenths:  $\frac{1}{10} = 0.1$ ,  $\frac{2}{10} = 0.2$ ,  $\frac{3}{10} = 0.3$ ,  $\frac{4}{10} = 0.4$ ,  $\frac{5}{10} = 0.5$ ,  $\frac{6}{10} = 0.6$ ,  $\frac{7}{10} = 0.7$ ,  $\frac{8}{10} = 0.8$ ,  $\frac{9}{10} = 0.9$

and reduce other equivalent fractions to their lowest terms.

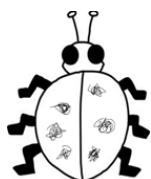
In the example above,  $3574 \div 8$ , children should be able to identify that the remainder as a fraction of  $\frac{6}{8}$  can be written as  $\frac{3}{4}$  in its lowest terms. As  $\frac{3}{4}$  is equivalent to 0.75, the answer can therefore be written as 446.75.

### Stage 1

Children are encouraged to develop a mental image of the number system in their heads to use for calculation. They should experience practical calculation opportunities involving **equal** groups and **equal** sharing.



They may develop calculations using A child's jotting spots between two



ways of recording pictures. showing halving six sides of a ladybird.

A child's jotting showing how they shared the apples at snack time between two groups.

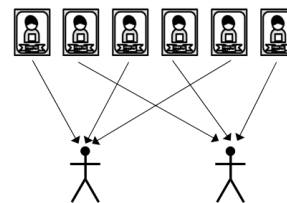


### Stage 2

Children explore practical contexts where they share equally and group equally.  $6 \div 2 = ?$

**Equal sharing (6 shared equally between 2)**

6 football stickers are shared equally between 2 people, how many do they each get? Children may solve this by using a 'one for you, one for me' strategy until all of the stickers have been given out.



**Equal grouping (How many groups of 2 are there in 6?)**

There are 6 football stickers, how many people can have 2 stickers each?



### Stage 3

Children continue to use practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation.

$12 \div 3 = ?$  Children begin to read this calculation as, 'How many groups of 3 are there in 12?'



At this stage, children will also be introduced to division calculations that result in remainders.

$13 \div 4 = 3$  remainder 1



### Stage 4

$43 \div 8$



$43 \div 8 = 5$  remainder 3

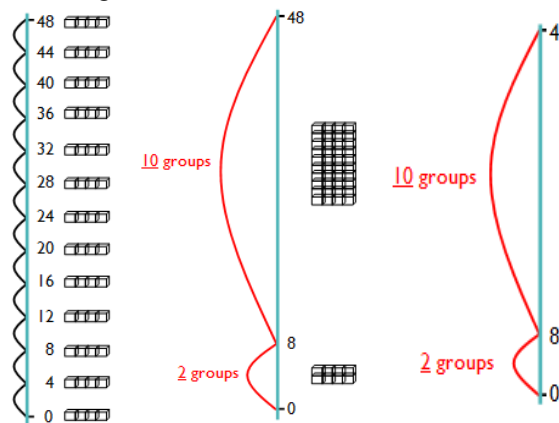
At this stage, children also learn if the remainder should be rounded up or down e.g.  $62 \div 8 = 7$  remainder 6

I have 62p. Sweets are 8p each. How many can I buy?  
Answer: 7 (the remaining 6p is not enough for another sweet)  
Apples are packed into boxes of 8. There are 62 apples. How many boxes do I need?  
Answer: 8 (the remaining 6 apples still need to be placed into a box)

### Stage 5

The previous method of repeated subtraction on a number line is continued, but using a vertical number line alongside practical equipment.

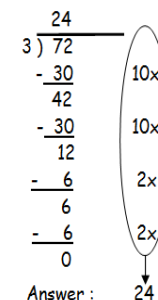
The repeated subtraction is made more efficient by subtracting 'chunks' of the divisor.



### Stage 6

This is the final stage, in which children use the 'chunking' method.

$72 \div 3$



1x	3
2x	6
5x	15
10x	30

Children should write key facts in a menu box. This will help them in identifying the largest group they can subtract in one chunk.

Stage 6 continued

$$196 \div 6$$

$$\begin{array}{r} 32 \text{ r } 4 \\ 6 \overline{) 196} \\ \underline{- 180} \\ 16 \\ \underline{- 12} \\ 4 \end{array}$$

30x  
2x

1x	6
2x	12
4x	24
5x	30
10x	60
20x	120

The key facts in the menu box should be extended to include 4x and 20x.

Answer : 32 remainder 4 or 32 r 4

Stage 6 continued

$$972 \div 36$$

$$\begin{array}{r} 27 \\ 36 \overline{) 972} \\ \underline{- 720} \\ 252 \\ \underline{- 252} \\ 0 \end{array}$$

20x  
7x

Answer : 27

Stage 6 continued

$$362 \div 17$$

$$\begin{array}{r} 21.29 \\ 17 \overline{) 362} \\ \underline{- 340} \\ 22 \\ \underline{- 17} \\ 5.0 \\ \underline{- 3.4} \\ 1.60 \\ \underline{- 1.53} \\ 0.07 \end{array}$$

To enable children to continue the calculation, they need to understand that 5 is the same as 5.0

20x  
1x  
0.2x  
0.09x

When recalling and deriving multiplication and division facts, children should also identify decimal equivalents of times tables,  
e.g. if  $2 \times 17 = 34$ , I know that  $0.2 \times 17 = 3.4$   
if  $9 \times 17 = 153$ ,  $0.9 \times 17 = 15.3$   
so  $0.09 \times 17 = 1.53$

Children should not be made to go onto the next stage if:

- 1) they are not ready.
- 2) they are not confident.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.